Vanishing Market Power ♦

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Abstract

Perfect competition can be approximated in an environment with differentiated goods, heterogeneous firms, and frictions of trading. This paper considers an environment where sellers sell differentiated goods to buyers, and frictions of trading are represented by the buyers having incomplete consideration sets of the sellers in the market. Besides selling differentiated products, some sellers are more "prominent" and so are present in a larger number of buyers' consideration sets. However, despite these imperfections, as the average number of sellers in the buyers' consideration sets expands, the sellers' market power vanishes and the equilibrium of the market approximates competitive conditions.

Keywords

Price formation, Imperfect competition, Frictions of trading, Perfect competition.

Resumo

A concorrência perfeita pode ser aproximada em um ambiente com bens diferenciados, empresas heterogêneas e fricções de mercado. Este artigo considera um ambiente onde vendedores vendem bens diferenciados a compradores, e as fricções de mercado são representados por conjuntos de consideração incompletos dos compradores com relação aos vendedores no mercado. Além de venderem produtos diferenciados, alguns vendedores são mais "destacados" e por isso estão presentes em conjuntos de consideração dos compradores. No entanto, apesar destas imperfeições, à medida que o número médio de vendedores nos conjuntos de considerações dos compradores se expande, o poder de mercado dos vendedores desaparece e o equilíbrio do mercado aproxima-se das condições competitivas.

Palavras-chave

Formação de preços, Concorrência imperfeita, Fricções de mercado, Concorrência perfeita.

Classificação JEL

C72, C78, D40, D83.

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1. Introduction

In most industries, firms sell differentiated products. It is often supposed that such a prevalence of product differentiation implies a prevalence of market power. However, economists have shown that even differentiated product markets can be highly competitive.[1] The degree of market power of a seller of a differentiated product depends on a variety of factors. For example, the supply capacity of the seller relative to the size of the market (Hart 1979), the number of competing sellers in the same product category (Perloff and Salop 1985), and the existence of search/sampling costs (Wolinsky 1986).

This paper generalizes these results by studying an environment where there are "frictions of trading," represented by limited access of the buyers regarding the set of sellers they can trade with. Buyers' imperfect access to the sellers is represented by buyers having incomplete consideration sets. This generalized method to describe frictions of access is consistent with various assumptions regarding buyers' access to different sellers.[2] This paper also allows some sellers to be included in the consideration sets of more buyers than others: this is a generalization of the concept of prominence (Armstrong, Vickers, and Zhou 2009), which expresses the idea that in a product category, some brands are more popular than others.

I study a market with a continuum of buyers and sellers. Each seller sells a particular brand and each buyer has unit demand but the buyer's valuations for different brands vary. The population of buyers that incorporates a seller into their consideration set is called the seller's "customer base," which varies across sellers.[3] On the other hand, the number of sellers that each buyer incorporates into her consideration set is distributed according to a Poisson distribution.[4] This paper also features an extension where buyers choose to search for the sellers that endogenizes this Poisson distribution in equilibrium.[5]

1 As Ostroy and Zame (1994) argued that the basic property of an economy so that competitive equilibrium is a plausible model is "thickness," which can be consistent with the existence of product differentiation.

2 For example, the assumption buyers have different search costs in papers like Stahl (1989) and Moraga-Gonzáles, Sándor, and Wildenbeest (2017), or that buyers have costs of switching from one seller to another as in Klemperer (1987).

3 As done in papers such as Allen, Clark, and Houde (2019) and Dinlersoz and Yorukoglu (2012).

4 As is Butters (1977), who assumed sellers' products are perfect substitutes.

5 Given the other buyers' search strategies and the profile of prices posted by the sellers, I show that there exists a distribution of search costs that implies that a buyer's optimal search strategy is consistent with such profile of customer bases.

The model features a unique symmetric equilibrium price posted by all sellers. As the mean number of sellers in a buyers' consideration set diverges to infinity (that is, as "frictions vanish"), then the equilibrium price converges to the perfectly competitive price. The environment studied has two possible cases: the competitive price can be either 1 or 0.

The first case, where the competitive price is 1, occurs if the seller's supply capacity is too low relative to demand. Then, as frictions vanish, the equilibrium price converges to the highest reservation price buyers might be willing to pay for any brand in the market. The second case, where the competitive price is 0, occurs if the seller's supply capacity is high enough, then the equilibrium price converges to the marginal cost. The reasoning is as follows: suppose a buyer i and seller j can realize a surplus v_{ij} from transacting the differentiated product. If buyer i has access to many other sellers, then any price higher than marginal cost implies that there is a high probability that i can access a competing seller j' and generate a surplus $v_{i,j'}$ that is very close to v_{ij}.

The first result, where sellers are on the short side of the market, corresponds to the convergence to perfect competition case studied in Hart (1979). The second result, where sellers are on the long side of the market, corresponds to the case studied in Wolinsky (1986).[6]

The paper is organized as follows: Section 2 describes the environment of the model and the pricing game among the sellers. Section 3 describes the solution concept, characterizes the competitive equilibrium, and shows that as frictions of trading vanish, the equilibrium of the model converges to the competitive equilibrium. Section 4 illustrates the results of this study solving a particular example of the model. Section 5 presents an extension of the model that endogenizes the consideration sets using a fixed sample search technology. Finally, Section 6 presents concluding remarks.

6 My results are also related to Lauermann (2013) regarding the assumptions so that the frictionless limits of random matching and bargaining games are competitive. In this case, I study an environment where the sellers have all the bargaining power.

2. Environment

Consider a market for a differentiated product with a continuum of buyers and sellers of unit measure. Each seller $j \in [0,1]$ enters the market with $q > 0$ units of a differentiated good of brand j. Each buyer has unit demand for the differentiated good with a valuation that varies across brands. Let v_{ij} be the valuation of buyer i regarding seller j's brand. The valuation v_{ij} is distributed according to a continuous cumulative distribution function (c.d.f.) G with support [0,1].

Note that the distribution of valuations for any brand among buyers is continuous; therefore, for any valuation $v < 1$, for the brand of the good offered by seller j, there is a positive measure of buyers with a valuation higher than v. This implies that for a seller j, we can index buyers in [0,1] with respect to their valuation of j's product with maximum valuation at $i = j$. Therefore, for z were

$$z = \begin{cases} i + (1 - j) & if\ i \leq j \\ i - j & if\ i > j \end{cases} \tag{1}$$

we have that

$$v_{ij} = G^{-1}(z), \tag{2}$$

for $i \in [0,1]$ (as G is a bijection from [0,1] to [0,1]). Note that $v_{ij} = 1$ if $i = j$. If G is uniform, the environment of this model is similar to the model of geographical differentiation of Hotelling (1929): sellers sell identical goods, but both buyers and sellers are uniformly distributed over a circle, and buyers have constant transportation costs. Section 4 characterizes the equilibrium of the model in such an environment.

Each seller j competes by posting a price p_j and buyers would like to shop at the highest surplus seller (that maximizes $v_{ij} - p_j$) as long as the posted price is lower than their reservation price. However, there are frictions of trading: buyers have constrained access to the sellers operating in the market.

2.1. Frictions of Trading: Constrained Access/Consideration Sets

Frictions of trading are represented by the buyers' constrained **consideration sets** regarding the sellers operating in the market.[7] Constrained consideration sets mean that a buyer i only considers a finite subset A^i of sellers from which she can purchase the product. Each seller $j \in [0,1]$ has a **customer base** $m^j \in \mathcal{R}_+$ which represents the relative population of buyers that incorporates seller j in their consideration set. Let

$$\widehat{m} = \int_0^1 m^j \, dj \tag{3}$$

be the average customer base. The ratio $m^j/\widehat{m}$ represents the relative prominence of j compared to other sellers (that is, how likely they are to incorporate that particular seller over other sellers in their consideration set).

Formally, access of buyers to the sellers (the sellers' consumer bases and the buyers' consideration sets) is represented by an assignment correspondence $\Phi: [0,1] \rightrightarrows [0,1]$ from sellers to buyers. This assignment correspondence Φ is defined as the limit of a sequence of assignment correspondences $\{\Phi_N\}_N$.

The assignment correspondence Φ_N is constructed as follows: consider a partitioning of the set of sellers into $N \in \mathbb{N}$ partitions $\{P_k\}_{k=1}^N$, each of measure $1/N$. The assignment correspondence Φ_N satisfies the following properties:

Assumption 1. Φ_N assigns buyers to at most one seller in each partition P_k; that is, formally, for a buyer i, the set $\Phi(i)_N^{-1} \cap P_k$ is either empty or contains one seller.

Assumption 2. The measure of buyers assigned to a seller in a partition P_k is $m_k/N \in [0,1)$. This means that sellers in partition P_k have a customer base of size m_k (thus, in this finite partition of sellers, there are at most only N types of sellers).

Assumption 3. We assume that buyers' accessibility regarding the sellers is randomly distributed. In the context of the assignment correspondence Φ_N, this assumption means that the fraction of buyers assigned to sellers in some partition P_k is independent of buyers assigned to sellers

[7] As in Armstrong and Vickers (2022).

in another partition P_k. That is, a fraction m_k/N of all buyers is assigned to partition P_k, and among the buyers assigned to sellers in $P_{k'}$, the same fraction, m_k/N, is also assigned to partition P_k.

As $N \to \infty$, then the partition of sellers becomes infinitely finer, and Φ_N is said to converge to a correspondence Φ that represents a profile of customer base sizes $\{m^j\}_{j\in[0,1]}$ if and only if the c.d.f. of customer base sizes described by Φ_N converges to the c.d.f. of customer base sizes of $\{m^j\}_{j\in[0,1]}$. Note that means that the sequence of assignment correspondences $\{\Phi_N\}_N$ enables us to approximate any desired distribution of sellers' customer base sizes.

2.1.1. A Simple Illustrative Example of The Distribution of Access

To more easily visualize how the frictions of trading function in this environment, consider a simple example: suppose a slightly different environment where there are only two sellers $j = 1,2$, and each seller has a customer base of 0.6. As access is random, the likelihood that a buyer has both sellers in her consideration set is $0.6^2 = 0.36$, and that a buyer does not have access to any seller is $0.4^2 = 0.16$. Note that buyers have, on average, 1.2 sellers in their consideration sets.

Suppose now that we replicate the number of sellers by N times, the number of sellers increases from 2 to $2N$. But, to keep the aggregate of customer bases of 1.2, an individual seller's customer base in this N-replicated market is $0.6/N$. Then, the cardinality of a buyers' consideration set is distributed according to a binomial distribution with $2N$ trials and a probability of success of $0.6/N$. As N increases to infinity, by the Poisson limit theorem, the distribution of the cardinality of the buyers' consideration set converges to a Poisson distribution with parameter 1.2.

2.1.2. The Distribution of Accessibility Regarding the Sellers is a Poisson Distribution

Lemma 1 below states that Assumptions 1-3 imply that the number of sellers that a buyer has in her consideration set is distributed according to a Poisson distribution with parameter $\widehat{m}$. As shown in Section 5, the asymmetry in the size of consideration sets can be made an endogenous equilibrium object through a fixed-sample search protocol with asymmetric search costs between buyers.

Lemma 1. *If Assumptions 1-3 are satisfied and accessibility is described by an assignment correspondence Φ, then the probability a buyer includes k different sellers in her consideration set is described by the Poisson probability mass function $\pi^k(\widehat{m})$ of k successes with parameter $\widehat{m}$. Additionally, the probability that a buyer has $k+1$ sellers in her consideration set conditional on being in some seller j's customer base is also $\pi^k(\widehat{m})$.*

Proof. The proof of this lemma and all of the following propositions and lemmas are in the Appendix: Proofs. □

2.2. The Pricing Game

Each seller chooses price $p \in \mathcal{R}$ to post to all buyers in his customer base. A strategy profile $s: [0,1] \to \mathcal{R}$ assigns to seller j a price $s(j)$ to be posted. Each buyer i observes the posted prices from the set of sellers in her consideration set and purchases the good at the highest surplus $v_{ij} - s(j)$ if this surplus is positive.[8]

Let $P(p,s)$ be the probability that a buyer purchases from seller j; it is a function of the posted price p and the strategy profile s of the other sellers. $P(p,s)$ satisfies

$$P(p,s) \quad = \pi^0(\widehat{m})P^0(p,s) + \pi^1(\widehat{m})P^1(p,s) + \pi^2(\widehat{m})P^2(p,s) + \cdots, \tag{4}$$

where $P^n(p,s)$ is the probability that a random buyer prefers j's "contract" (the price and brand combination (p,j)) over both the contracts posted by n competing sellers and over the possibility of not trading. $\pi^n(\widehat{m})$ is the probability that a buyer that has access to j also has n other sellers in her consideration set. See the Appendix, subsection Derivation of P, for the characterization of P^n and $P(p,s)$.

[8] The structure of the game is similar to models such as Perloff and Salop (1985) and Ivanov (2013), but with buyers having imperfect access to the brands available and with this accessibility being private information.

As each seller has a customer base with a continuum of buyers, his profits are deterministic and given by

$$\Pi_j(p,s) = p\min\{m^j P(p,s), q\}, \quad (5)$$

where q is the seller's endowment of the good.

3. Equilibrium

The equilibrium notion of the game is Nash equilibrium: a profile s^* of prices posted by the sellers such that, for each seller j, posting a price $s^*(j)$ maximizes profits given that the other sellers are posting prices according to s^*.

Definition 1. An equilibrium is a strategy profile $s^*: [0,1] \to \mathbb{R}_+$ such that for each seller j, posting $s^*(j)$ is a best response to s^*. That is, $s^*(j)$ maximizes $\Pi_j(p, s^*)$ over p given that all other sellers play according to strategy profile s^*.

3.1. Convergence to Competitive Equilibrium

The main result of this paper is that as frictions of trading vanish (which here means that the buyers' consideration sets expand to include many firms), the posted prices and the allocation of the strategic equilibrium converge to the prices and the allocation of the perfectly competitive equilibrium. However, we first must define and characterize the competitive equilibrium for this environment so we can compare it with the strategic equilibrium of the price competition game: a competitive (frictionless) equilibrium is a profile of prices for each brand j such that demand for each brand j is equal to supply and the market is frictionless (i.e., buyers have full and costless access to all sellers present in the market).

3.1.1. Definition and Characterization of the Competitive Equilibrium

A competitive equilibrium features a single price for each brand. In this environment, a profile of prices for brands is described by a function $p: [0,1] \to \mathbb{R}_+$.

Although there is a continuum of buyers in this economy, there is only one buyer i with preferences given by the valuation function v_{ij}, and i's demand is indivisible. Therefore, to make the demand correspondence convex, we allow buyers to randomize their consumption. For example, a buyer i can consume brand l with probability α and not consume any brand with probability $1-\alpha$.

Let

$$\Delta = \{x \in \mathcal{F}([0,1]) : \textstyle\sum_{k\in[0,1]} x\,(k) \leq 1\}, \tag{6}$$

where $\mathcal{F}([0,1])$ is the set of functions mapping $[0,1]$ into itself, be the consumption set of buyers. A $x \in \Delta$ is a function that assigns a positive probability of consuming brands over a finite subset of different brands and probability zero for all other brands. Note that if $\sum_{k\in B} x\,(k) < 1$, then the buyer assigns a positive probability of not consuming any brand.

The buyer's problem is to choose a finite subset of brands that maximize her utility. For buyer i, the bundle $x_i \in \Delta$ is consistent with Walrasian demand if and only if it satisfies

$$x_i \in \underset{x\in\Delta}{argmax} \textstyle\sum_{\{j:x_i(j)>0\}} x\,(j)[v(i,j) - p(j)] \tag{7}$$

Let X be the aggregate demand profile implied by a profile of individual consumption choices $(x_i)_{i\in[0,1]}$. It satisfies for $j \in [0,1]$ and $p: [0,1] \to \mathbb{R}$,

$$X(j,p) = \begin{cases} \sum_{\{i:x_i(j)>0\}} x_i\,(j) & \text{if } |\{i: x_i(j) > 0\}| < \infty \\ \infty & else. \end{cases} \tag{8}$$

Seller j's supply correspondence $Y(j, p^W)$ of brand j satisfies

$$Y(j,p) = arg \underset{z\in[0,q]}{max}\, p(j)z \tag{9}$$

The pair $\{(x_i)_{i\in[0,1]}, Y(:,p)\}$ represents an allocation of demand profiles $(x_i)_{i\in[0,1]}$ and a function $Y(:,p)\colon [0,1] \rightrightarrows [0,q]$ that specifies the quantities supplied by each seller.

Definition 2. A competitive equilibrium is an allocation $\left((x_i)_{i\in[0,1]}, Y\right)$ and a profile of prices $p^W\colon [0,1] \to \mathbb{R}_+$ such that $X(l,p^W) \in Y(l,p^W)$ for all brands/sellers $j \in [0,1]$.

Proposition 1. *The competitive equilibrium price* $p^W\colon [0,1] \to \mathbb{R}_+$ *is*

$$p^W(j) = \begin{cases} 0 & \text{if } q > 1 \\ [0,1] & \text{if } q = 1 \\ 1 & \text{if } q < 1 \end{cases}$$

for any brand $j \in [0,1]$.

To understand Proposition 1, first consider the case where $q > 1$. Suppose that $p^j > 0$ for every brand j, then the supply of every brand is $q > 1$ as sellers have zero marginal cost. Since buyers have unit demand, demand for at least some brands will be less or equal to 1. At negative prices, no seller would be willing to supply the good, while demand will be greater than zero, at least for some brands. If $p(j) = 0, \forall j$, the supply correspondence for each brand satisfies $Y(j,p) = [0,q]$ and demand for each brand is $1 \in [0,q]$, hence, it is an equilibrium.

For the case where $q = 1$, any price $p \in [0,1]$ for all brands is consistent with equilibrium: each buyer i is willing to purchase one unit of brand i (as they are satiated), and each seller is willing to supply one unit of his particular brand. However, if prices are not symmetric across different brands, then such a profile of prices might not be consistent with equilibrium as infinitely many buyers might be interested in some particular brand.

Finally, in the case where $q < 1$, if prices are higher than 1 for some brand j, then the demand for j will be zero, and the supply will be $q > 0$. Which is not consistent with equilibrium. If prices are lower than 1 for a positive measure of brands, then demand for those brands will be higher than supply, which is also not an equilibrium. For prices $p(l) = 1, \forall l$, a demand of q for each brand l is consistent with utility maximization, and supply for each brand is q. Hence, it is an equilibrium.

3.1.2. Convergence of the Strategic Equilibrium to the Competitive Equilibrium

Note that buyers' reservation prices are always bounded above by 1 and the sellers' costs are always zero. Therefore, if $q = 1$, then strategic equilibrium prices are trivially consistent with the competitive equilibrium prices. However, if the size of the consideration sets of a subset of buyers of measure one diverges to infinity, then, as stated in the Theorem below, for any q we have convergence in regards to the equilibrium prices to the competitive equilibrium.

Proposition 2. *Consider a sequence $\{\mathbf{m}_n\}_n$ of seller customer base profiles $\mathbf{m}_n = \left(m_n^j\right)_{j\in[0,1]}$ such that*

$$\lim_{n\to\infty} m_n^j = \infty$$

for almost all $j \in A$. Then, first the equilibrium pricing strategy $s^*(j, \mathbf{m}_n)$ must satisfy for almost every $j \in [0,1]$

$$\lim_{n\to\infty} s^*(j, \mathbf{m}_n) = \begin{cases} 1 & \text{if } q < 1 \\ 0 & \text{if } q > 1 \end{cases}$$

Second, the equilibrium allocation converges to a competitive equilibrium allocation $\left((x_i)_{i\in[0,1]}, Y\right)$. This convergence occurs in the sense that for each buyer i, the brand j purchased by i in equilibrium converges to $j^* = i$, which is the brand consumed in competitive equilibrium, and the quantity consumed converges to a competitive equilibrium quantity.

Proposition 2 states that the strategic equilibrium converges to the competitive equilibrium as $\widehat{m}_n$ increases to infinity. That is, as frictions vanish, individual buyers have access to many sellers, and the market's "network" becomes perfectly "thick", then competitive behavior emerges as long as the difference in seller customer base size remains bounded.

The reasoning behind the proof is as follows: first, consider the case where $q > 1$. Suppose that all sellers are posting a price p^*. Consider a buyer i who has some arbitrary seller j in her consideration set and has valuation $v_{ij} < 1$ for j's brand. Note that the probability that buyer i incorporates in her consideration set another seller j' who is selling a brand which she values at least as much as $v_{ij} + \epsilon$, for any $\epsilon \in \left(0, 1 - v_{ij}\right)$, converges to 1 as $\widehat{m}_n \to \infty$. Therefore, the elasticity of j's demand diverges to infinity as $\widehat{m}_n \to \infty$. This means that the logic of Bertrand competition applies: it becomes profitable for any seller to undercut the price p^* if his supply constraint is not binding. Therefore, as $\widehat{m}_n \to \infty$ the equilibrium pricing strategy $s^*: [0,1] \to \mathbb{R}_+$ must converge to 0 almost everywhere if $q > 1$.

Now, consider the case where $q < 1$. Then $\widehat{m}_n \to \infty$ implies that the measure of j's customer base that has a valuation that is at a neighborhood of 1 diverges to infinity and, therefore, demand for a seller's product converges to 1 for a symmetric price posted by sellers that is strictly smaller than 1. Given that the sellers operate under a supply constraint $q < 1$, then feasibility implies that the equilibrium prices must converge to 1 as the customer bases grow to infinity for almost every seller.

The convergence result of Proposition 2 is not valid if a positive measure of customer bases does not diverge to infinity even if $\lim \widehat{m}_n = \infty$. Consider the following example:

$$m_n^j = \begin{cases} n & \text{if } j \in [0,.1] \\ 1 & \text{if } j \in (.1,1] \end{cases}$$

and $q \in \big(1, 10\exp(-.9)\big)$. As $q > 1$, the competitive equilibrium price is zero. However, suppose that all sellers post a price $p \in (0,1)$. As $n \to \infty$ the demand for the brand of a seller $j \in [0,.1]$ increases to a quantity that is greater than $10\exp(-.9)$. This occurs because, given symmetric prices, buyers shop at their favorite brands and a fraction $\exp(-.9)$ of the buyers lack access to a seller $j' \in (.1,1]$. Then, as $10\exp(-.9)$ is greater than 1, given the symmetric pricing strategy p, as n diverges to infinity, seller j's optimal pricing strategy is to post a price that converges to 1 and not to 0.

3.2. Existence of the Strategic Equilibrium

This subsection presents a set of sufficient (albeit highly restrictive) conditions for the existence of the equilibrium of the pricing game among the sellers. If either all the seller's customer bases are smaller than their supply capacity q or if the sellers' customer bases are the same (which is the case in the standard search models: matching probability is uniform and there is no prominence) then there is a unique equilibrium strategy profile that features a single price posted by all sellers p^*.

This is a result of product differentiation: Butters (1977) studied an environment without product differentiation with a similar accessibility technology (but without the asymmetry regarding customer base sizes/prominence), and the equilibrium always featured mixed pricing strategies.[9] The reason is that the inclusion of product differentiation leads to a purification of the equilibrium: under Bertrand competition, demand is infinitely elastic when one firm undercuts the other, but when the seller's products are not perfect substitutes for each other, then the demand curves facing the firms become continuous.

Proposition 3. *If G is concave and sellers' customer bases satisfy $m^j \leq q, \forall j$ or $m^j = \widehat{m}, \forall j$ or then there exists a price $p^* \in [0,1]$ such that posting p^* is a best response by each seller to the strategy profile $s^*(j) = p^*, \forall j$.*

4. An Example

To illustrate these results, consider the following example: following the model of geographical differentiation in Hotelling (1929), the closed unit interval [0,1] can represent locations in a circle, buyers prefer brands closer to their location, and in this case, G represents the linear "transportation" costs. Let

$$\delta(i,j) = min\{1 - |i - j|, |i - j|\} \tag{10}$$

be the distance between i and j in [0,1]. Transportation costs are

$$\lambda = 2(1 - v_L) \tag{11}$$

where $v_L \geq 1/2$. Then, $(1 - x)/[2(1 - v_L)]$ is the distance between a seller and a buyer in the circle if the buyer's valuation for the seller's good is x. Suppose that all sellers have the same customer base m.

Note that the monopoly price is $\overline{p} = v_L$. Therefore, all buyers prefer to trade at the monopoly price rather than not trade. Suppose that a seller j considers posting the price $p \leq \overline{p}$ and that the other sellers are following a symmetric strategy, posting price p'. Let ϵ given by

$$\epsilon = \lambda(p' - p) \tag{12}$$

[9] A result that is also present in many papers, such as Burdett and Judd (1983) and Varian (1980).

The term ϵ represents the undercutting factor, which is the magnitude of the difference in the price posted by j and the prices posted by the other sellers relative to the degree of product substitutability.

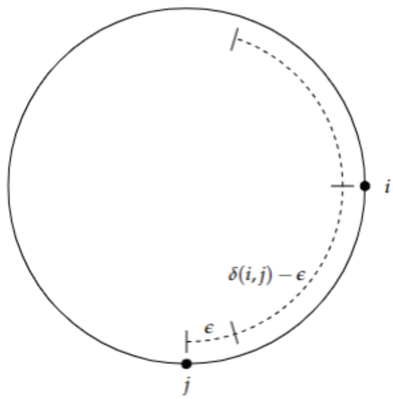

Figure 1 - If the distance between buyer i and seller j' is larger than $\delta(i,j) - \varepsilon$, then buyer i prefers to shop at j over j'.

As shown in Figure 1, the geometry of the circle implies that the probability that i prefers j over a competing brand j' is

$$\int_0^1 I_{\{v_{ij}-v_{ij'}\geq -\epsilon/(1-v_L)\}}(i,j')dj' = [1-2(\delta(i,j)-\epsilon)] \cap [0,1] \tag{13}$$

Therefore, the probability that a buyer purchases from seller j conditional on having $n \geq 0$ other sellers in her consideration set is

$$\begin{aligned} P^n(p,p') &= \int_0^1 \{[1-2(\delta(i,j)-\epsilon)] \cap [0,1]\}^n \, di \\ &= \begin{cases} \frac{1}{n+1}(1-2\epsilon)^{n+1} + 2\epsilon & if \, \epsilon > 0 \\ \frac{1}{n+1}(1+2\epsilon)^{n+1} & if \, \epsilon < 0. \end{cases} \end{aligned} \tag{14}$$

Substituting in Equation 5 implies that P is given by

$$P(p,p') = \begin{cases} 2\epsilon + \frac{1}{a}[exp(-2\epsilon m) - exp(-a)] & if \, \epsilon > 0 \\ \frac{1}{m}[exp(2\epsilon a) - exp(-a)] & if \, \epsilon < 0. \end{cases} \tag{15}$$

The proposition below states that the symmetric Nash equilibrium exists if $\hat{m} \notin [m_L, m_H]$ and it is unique. Where m_L and m_H are positive real numbers such that $m_L < m_H$.

Proposition 4. *If buyer valuations for each brand are distributed according to G there is a pair $m_L, m_H \in \mathbb{R}_+$ where $0 < m_L < m_H$, such that if m satisfies $m \notin [m_L, m_H]$ then there exists a corresponding unique symmetric Nash Equilibrium price $p^* \in (0,1]$.*

The quantity sold by each seller in equilibrium is

$$min\{q, 1 - \pi^0(m)\} \tag{16}$$

If $q \geq 1 - \pi^0(m)$, p^* is given by

$$p^* = min\left\{\overline{p}, \frac{\lambda}{2m}\right\} \tag{17}$$

If the supply constraint is binding ($q < 1 - \pi^0(m)$), the equilibrium price is

$$p^* = 1 - \frac{\lambda}{m}\{log[1 - exp(-m)] - log[1 - exp(-m) - q]\} \tag{18}$$

Note that an equilibrium does not exist for some m. That is because $P(p', p)$ is not always concave over [0,1], so Proposition 3 does not apply.

4.1. Numerical Illustration

Consider the case where $\lambda = .5$ then $[m_L, m_H] = [0.50, 0.72]$. For a customer base $m \notin [0.50, 0.72]$ and $q > 1$, then $p^*(m)$ satisfying Equation 17 is the unique equilibrium. If sellers have a customer base $\mathrm{m} \in [0.50, 0.72]$, then a symmetric pure-strategy equilibrium does not exist. Figure 2 plots the equilibrium price as a function of . Note that it converges to 0 as diverges to infinity.

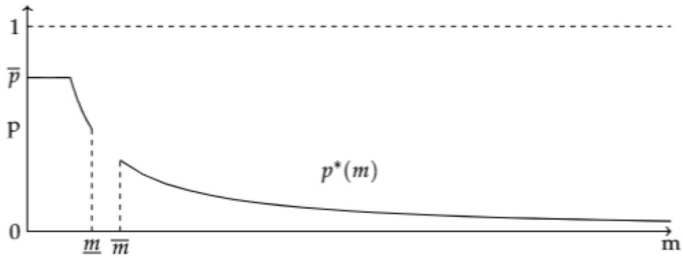

Figure 2 - Equilibrium price as a function of m with a non-binding supply constraint.

Suppose that the supply constraint is smaller than . For example, $q = 3/4$, then for $a > 1.39$, the supply constraint is binding, so the equilibrium price jumps from 0.18 to 0.75 and converges to 1 as $m \to \infty$. This discontinuity in price posting arises when is high enough for the supply constraint to bind. In this case, sellers cannot undercut their prices because they lack the supply capacity to meet the additional demand, as depicted in Figure 3.

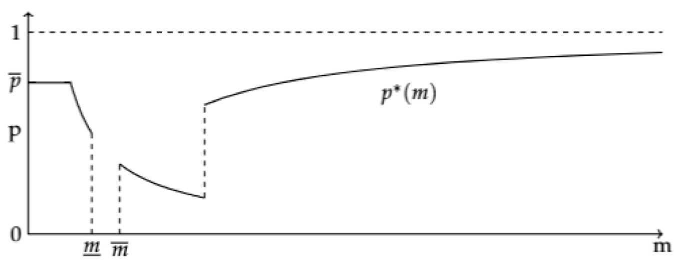

Figure 3 - Equilibrium price as a function of m with a binding supply constraint at $m > 1.39$.

5. Extension: Endogenous Accessibility as a Result of Buyers' Search Strategies

Consider an extension of the model from an environment where buyers' consideration sets are exogenous to an environment where they choose how many sellers to incorporate in their consideration sets. I consider a fixed sample search algorithm where buyers choose how many sellers to search for and incorporate into their consideration sets. Search is random and the likelihood of sampling is assumed to be in proportion to a seller's customer base.

The lemma below states that if average prices are low enough, then there exists a search cost c^k such that for a buyer i choosing to incorporate k sellers into her consideration set is optimal.

Lemma 2. *Suppose that sellers follow strategy s such that $\int m^j s(j)dj/\widehat{m} < 1$, then for $k \in \{0,1,\dots\}$ there exists a search cost $c^k > 0$ such that sampling k sellers is optimal.*

The proposition below, which follows from Lemma 2, states that if average prices are low enough, then there exist search costs distributed across the buyers in the economy, such that a distribution of search sample sizes according to a Poisson distribution with parameters equal to the sellers' average customer base $\widehat{m}$.

Proposition 5. *Given customer base profile* $\mathbf{m}$ *and a pricing strategy s such that $\int m^j s(j)dj/\widehat{m} < 1$ then there exists a distribution of search costs among buyers represented by a Poisson probability mass function π with parameter $\widehat{m}$, where $\pi(k)$ is the proportion of customers with search costs $c^k > 0$, and the profile of search costs $\{c^k\}_k$ is such that each customer finds it optimal to search for k different sellers.*

6. Concluding Remarks

This paper presented a model of price formation under imperfect competition that generalizes and extends results from the existing literature. It considers an environment where there are frictions of trading represented by incomplete and asymmetric buyer's consideration sets: different buyers can transact with different numbers of sellers and different sellers have different degrees of prominence (that is, buyers are more likely to have some sellers in their consideration sets than others).

This study showed that as frictions of trading decrease (which means buyers incorporate more sellers in their consideration sets), the imperfectly competitive equilibrium approximates the competitive equilibrium.

Therefore, even in markets where different sellers sell differentiated products and some sellers are relatively more prominent than others, if frictions of trading are low and buyers and sellers are small relative to the market, then such markets still can be accurately described by the model of perfect competition.

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Appendix: Proofs

Proof of Lemma 1

Proof. For simplicity, suppose that all sellers have the same customer base size, $m^j = m > 0$. Therefore, for an assignment correspondence Φ_N, the set of buyers assigned to sellers in a partition P_k has measure m/N.

Take a buyer i randomly and suppose that i incorporates some seller $j \in P_k$ in her consideration set (conversely, j incorporates i in his customer base); then Assumption 3 implies that the probability that i has a seller from P_k in her consideration set is m/N. Therefore, we have the following:

(1) The distribution of the number of sellers that i incorporates in her consideration set is a Binomial distribution with N trials and a probability of success of m/N.

(2) The distribution of the number of sellers besides j that i incorporates in her consideration set is a Binomial distribution with $N-1$ trials and a probability of success of m/N.

Note that $\frac{m}{N} \times N = m$ for all N. Therefore, taking to infinity implies that the Poisson Limit Theorem applies for (1), and therefore, the distribution of the number of sellers that i incorporates in her consideration set converges to a Poisson distribution with parameter m. In addition, as $\frac{m}{N} \times (N-1) \to m$ as N diverges to infinity, the Poisson Limit Theorem also implies to the distribution of the number of sellers besides j that i incorporates in her consideration set. In addition, it also converges to the same Poisson distribution with parameter m.

To extend the result to cases where sellers' customer bases vary, it suffices to consider the case where half of the sellers have a customer base of m_1 and the other half a customer base of m_2 (let's call them types 1 and 2). Consider the partition of sellers into $2N$ partitions of measure $1/2\,N$ and the first N partitions have sellers with customer base m_1, and the last N partitions have the sellers with customer bases of m_2.

Then, we can apply the previous reasoning to conclude that as N grows large, then the distribution of the sizes of buyers' consideration sets converges to (1) a Poisson distribution with parameter $m_1/2$ for the number of sellers of type 1 that buyers might have access to. (2) a Poisson distribution with parameter $m_2/2$ for the number of sellers of type 2 that buyers might have access to. Note that the distribution of the sizes of buyers' consideration sets then converges to a Poisson distribution with parameter $\widehat{m} = m_1/2 + m_2/2$. It is clear that this reasoning can be applied to any finite set of sellers' types. □

Derivation of a closed form of *P*

Buyers are distributed uniformly over the unit interval, so $P^n(p, s)$ is given by

$$P^n(p,s) = \int_{[0,1]^n\times[0,1]^{n+1}} I_{\{x-p\geq max\{\left(x^k-s(j^k)\right)_{k=1}^n,0\}\}}(x,x^1,j^1,\dots,x^n,j^n)d[G(x),G(x^1),F(j^1),\dots,G(x^n),F(j^n)], \quad (19)$$

where

$$I_{\{x-p\geq max\{\left(x^k-s(j^k)\right)_{k=1}^n,0\}\}}(x,x^1,j^1,\dots,x^n,j^n) \in \{0,1\} \quad (20)$$

is the indicator function of whether the contract (p, j) is preferred by i over not trading (i.e., $x - p \geq 0$), and over the profile of contracts $\left(s(j^k), j^k\right)_{k=1}^n$ posted by sellers $j^1, \dots, j^n$ (i.e., $v_{ij} - p \geq v_{ij^k} - s(j^k), \forall k \in \{1, \dots, n\}$) given the profile of valuations $(x, x^1, \dots, x^n)$, of j's and j's n competitors products.

The indicator function satisfies

$$\begin{aligned} & I_{\{v_{ij}-p\geq max\{\left(v_{ij^k}-s(j^k)\right)_{k=1}^n,0\}\}}(i,j^1,\dots,j^n) \\ &= I_{\{x-p\geq 0\}}(x) \times I_{\{x-p\geq x^1-s(j^1)\}}(x,x^1,j^1) \times \dots \times I_{\{x-p\geq x^n-s(j^n)\}}(x,x^n,j^n), \end{aligned} \quad (21)$$

therefore, Equation 19 and the Fubini–Tonelli theorem implies that $P^n(p, s)$ satisfies

$$\begin{aligned} & P^n(p,s) \\ &= \int_0^1 I_{\{x-p\geq 0\}}(x) \begin{bmatrix} \int_0^1 \left(\int_0^1 I_{\{x-p\geq x^1-s(j^1)\}}(x,x^1,j^1)dF(j^1)\right) dG(x^1) \times \\ \dots \times \int_0^1 \left(\int_0^1 I_{\{x-p\geq x^n-s(j^n)\}}(i,j^n)dF(j^n)\right) dG(x^n) \end{bmatrix} dG(x) \\ &= \int_0^1 I_{\{x-p\geq 0\}}(x) \left[\int_0^1 \left(\int_0^1 I_{\{x-p\geq x^1-s(j')\}}(x,x',j')dF(j')\right) dG(x')\right]^n dG(x). \end{aligned} \quad (22)$$

Substituting $P^n(p,s)$ in Equation 4 by the bottom left-hand side of Equation 22 implies that $P(p,s)$, the probability of sale to an arbitrary buyer, satisfies

$$P(p,s) = \int_0^1 I_{\{x-p\geq 0\}}(x)exp\left[-\hat{m}\left(1-\int_0^1\int_0^1 I_{\{x-p\geq x'-s(j')\}}(x,x',j')dF(j')dG(x')\right)\right]dG(x) \quad (23)$$

Proof of Proposition 1

Proof. First, we need to check whether the prices specified by the proposition implement a competitive equilibrium. Then, we see that they are the only prices that implement the equilibrium.

Existence: Let $q < 1$. Suppose that $p^W(l) = 1$ for all brands l. With a price equal to 1 for all brands, only buyer $i = j$ will be willing to purchase good j as any $i \neq j$, $v_{ij} < 1$.

Therefore, demand for each brand l will be [0,1], as buyer $i = j$ will be almost always indifferent between purchasing brand i or not (randomizing between the two); thus, demand for each brand is [0,1], while all sellers will supply $q \in (0,1)$ as prices are strictly greater than zero. Therefore, it is an equilibrium.

Let $q > 1$, if $p^W(l) = 0$ if for all l then demand for each brand l will be 1, as almost every buyer $i \in [0,1]$ will purchase a unit of brand $i \in [0,1]$, while supply is given by $S(l) = [0,q]$. Therefore, it is an equilibrium.

Uniqueness: It remains to show that equilibrium is unique. First, consider the case $q < 1$; we need to check that any pricing profile different from $p^W(l) = 1$ for all $l \in [0,1]$ inconsistent with equilibrium.

Consider a profile of prices p^W with $p^W(l) < 1$ for some subset of products $Z \subset [0,1]$ and $p^W(j) = 1, \forall j \notin/Z$. Then, continuity of v_{ij} in i (from [eq: v_ij]) implies that for some buyers i near Z will find brands in Z that yield higher utility than not-trading. Therefore x_i is such that $\sum_{j\in Z} x_i(j) = 1$, then it is easy to see that $X(l) \geq 1$ for at least some $l \in Z$, a contradiction with equilibrium.

Second, consider the case where $q > 1$. Suppose the prices are such that $p^W(l) > 0$ for subset of brands $Z \subset [0,1]$ and $p^W(j) = 0$. As prices are strictly greater than 0, demand for some brands in Z is, at most, 1, as buyers will not want to purchase more than one unit of the consumption good, and if multiple buyers purchase the same brand, other brands in Z will have demand less than 1, but the supply of any brand in Z will be equal to $q > 1$, a contradiction. □

Proof of Proposition 2

Proof. Part 1: *Convergence in prices*

First note that $m_n^j \to \infty$ for every $j \in [0,1]$ implies that $\widehat{m}_n = \int_0^1 m_n^j \, dj$ converges to infinity. Consider a buyer $i \in [0,1]$, as $n \to \infty, \widehat{m}_n \to \infty$ thus the expected number of sellers in i's consideration set diverges to infinity. The continuity of G and independence of valuations of different brands implies that the expected valuation of i's preferred brand converges to 1. That is, as buyers incorporate more brands in their consideration sets, the expected valuation of the preferred brand converges to 1. Also note that in equilibrium, $s^*(j, \mathbf{m}_n) \in [0,1), \forall j, \forall n$, as negative pricing is never consistent with equilibrium: it is easy to see that a seller post negative prices then its profits will be strictly negative since $\pi^0(m) > 0$ for any $m \in \mathbb{R}_+$ and pricing above all buyer's reservation price yields zero profits.

The proof proceeds by contradiction. Consider the case where $s^*(j, \mathbf{m}_n)$ converges to a symmetric pricing profile, that is $\lim_{n\to\infty} s^*(j, m_n) = p^*$ for all brands j. In an equilibrium where all sellers post the same price, buyers will purchase their preferred brand. Therefore, the reservation price of the customers of a seller j converges in probability to 1. This implies that the population of buyers who purchase some brand also converges to 1 if $\lim_{a\to\infty} s^*(j, \mathbf{m}_n) < 1$. This implies that the average quantity sold by sellers converges to 1 in such equilibrium if the equilibrium price converges (taking a subsequence if necessary) to a number that is less than 1.

Consider the case where $q > 1$ and suppose that $p^* > 0$. Note that the average quantity sold in equilibrium by sellers converges to 1, which means that for some seller j, the quantity sold converges to a quantity equal or smaller than 1 as $\widehat{m}_n \to \infty$, which is below supply constraint q. Suppose that a seller j considers reducing the posted price p^* by some undercut factor $\epsilon > 0$ while all other sellers post p^*. Then, equation [eq: analytical form] implies that quantity sold is given by

$$\begin{aligned} & m_n^j P_n(p^* - \epsilon, p^*) \\ & = m_n^j \int_0^1 I_{\{x \geq p^* - \epsilon\}}(x) exp\left[-\widehat{m}_n \left(1 - \int_0^1 I_{\{x \geq x' - \epsilon\}}(x, x') dG(x')\right)\right] dG(x) \\ & = m_n^j \int_0^1 I_{\{x \geq p^* - \epsilon\}}(x) exp\left[-\widehat{m}_n \left(1 - \left(\int_0^1 \left[I_{\{x \geq x'\}}(x, x') + I_{\{x \in [x' - \epsilon, x')\}}(x, x')\right] dG(x')\right)\right)\right] dG(x) \end{aligned} \tag{24}$$

where P_n has the n subscript as it changes for each $n \in \mathbb{N}$. As $p^* - \epsilon < 1$ the continuity of G implies that $v_{ij} > p^* - \epsilon$ for all i in some neighborhood of j, and that the set $\{j' \in [0,1]: v_{ij} \in [v_{ij'} - \epsilon, v_{ij'})\}$ has strictly positive measure if $i \neq j$. Therefore, $P_n(p^* - \epsilon, p^*) > P_n(p^*, p^*)$. In words, the quantity sold increases for an $\epsilon > 0$ cut in price: as there is a population of buyers in j's customer base who prefer some other brand j' among the $\widehat{m}$ different brands they have access to but the difference in reservation price between those other brands, and j is smaller than ϵ.

Note that increasing n to infinity, as implied by expression [eq: change in quantity sold-1], implies that the elasticity of the quantity sold to a decrease in the posted price diverges to infinity. Therefore, for n large enough, seller j has an incentive to undercut the competition, as the increase in sales is larger than the decrease in profit margin, a contradiction with p^* being equilibrium. Therefore, $s^*(j, \mathbf{m}_n)$ does not converge to a price higher than 0. Therefore, if the Nash equilibrium strategy profile converges to a symmetric pricing profile, it will converge to the price of 0 for every brand if the sellers are endowed with $q > 1$ units of the consumption good.

Conversely, consider the case where $q < 1$ and suppose that $p^* < 1$. Note that as the average quantity sold in equilibrium by sellers converges to 1 in symmetric equilibrium if $p^* < 1$ then the demand for some seller j posting $s^*(j, \mathbf{m}_n)$ converges to a quantity greater or equal than 1 as $n \to \infty$, which is above supply capacity. Continuity of v implies demand (that is $m_a^j P_n(p, s, \widehat{m})$) is continuous. Thus, sellers can increase profits by increasing prices, a contradiction with $s^*(j, \mathbf{m}_n)$ being Nash equilibrium. This implies that as $\widehat{m}_n$ increases to infinity, any price below 1 is ruled out in equilibrium. Since $p^* \in [0,1]$, therefore, if the Nash equilibrium strategy profile converges to a symmetric pricing profile, it will converge to the price of 1 for every brand.

It remains to show that the case where $s^*(j, \widehat{m})$ does not converge to a symmetric pricing profile almost everywhere is not consistent with equilibrium.

Case 1: Suppose that $q > 1$. Consider the case where for a positive measure of brands $Z \subset [0,1]$ with measure smaller than 1, $s^*(j, \mathbf{m}_n)$ does not converge to 0, $\forall j \in Z$. Then (taking subsequences if necessary) $\{s^*(j, \mathbf{m}_n): j \in Z\}$ converges to a profile of prices $s^*(j) > 0, \forall j \in Z$. Since Z has a positive measure, there is a price $p > 0$ such that the set $Z' = \{j \in Z: s^*(j) \geq p\}$ has a positive measure. As $m_n^j \to \infty$ for sellers j in Z' and as Z' has positive measure then $\widehat{m}_n(Z') = \int_{j \in Z^j} m_n^j \, dj$ converges to infinity. Then, by analogous argument, as in the symmetric pricing case (from Equation 24), a small undercut by seller $j \in Z'$ of the price p will yield a marginal log increase in quantity sold (drawn from customers of other sellers in Z') that increases to infinity as $n \to \infty$. Which implies that for a seller $j \in Z'$ posting a price $p' < p \leq s^*(j)$ yields higher profits than posting $s^*(j)$ for n large enough. A contradiction with $s^*(j, \mathbf{m}_n)$ being Nash equilibrium for every n.

Case 2: Suppose that $q < 1$. Consider the case where, for a positive measure of brands, $Z \subset [0,1]$, $s^*(j, \mathbf{m}_n)$ does not converge to 1, $\forall j \in Z$. Then (taking subsequences if necessary) $\{s^*(j, \mathbf{m}_n): j \in Z\}$ converges to a profile of prices $s^*(j) < 1, \forall j \in Z$. Since Z has a positive measure, there is a $p < 1$ such that the set $Z' = \{j \in Z: s^*(j) \leq p\}$ has a positive measure.

As $m_n^j \to \infty$ for sellers j in Z' and as Z' has a positive measure then $\widehat{m}_n(Z') = \int_{j \in Z^j} m_n^j \, dj$ diverge to infinity. This implies that for an arbitrary buyer i, the likelihood that i includes a seller $j \in Z'$ in her consideration set converges to 1 as $n \to \infty$.

Note that for any seller j in Z', since $s^*(j) \leq p < 1$, there is a strictly positive measure of buyers, $1 - G\big(s^*(j)\big)$, who have a strictly higher valuation in shopping at j than at any sellers outside of Z' (because those sellers post prices higher than p). Therefore, as accessibility improves with a larger n ($\widehat{m}_n(Z')$ diverge to infinity), buyers who prefer brands from sellers outside of Z' choose to shop at sellers in Z' because the prices are lower. That implies that as $n \to \infty$, the measure of buyers who choose to shop at sellers in Z'converges to a number larger than the measure of the set of sellers Z'.

Thus, as $n \to \infty$, for at least some sellers in Z, demand for their output will exceed 1 (as the demand of each buyer with a valuation higher than the posted price is 1), but their supply capacity is bounded by $q < 1$. Continuity of G implies that demand (that is $m_a^j P_n(p,s)$) is continuous. Therefore, such sellers in Z can increase profits by increasing prices, a contradiction with $s^*(j, \mathbf{m}_n)$ being a Nash equilibrium.

Part 2: *Convergence in allocation*

To see that the equilibrium allocation converges for almost every brand to the competitive equilibrium allocation, first, let us consider the sellers. As equilibrium sales converge to 1 almost everywhere if $q > 1$, and to q if $q < 1$, then quantities supplied in equilibrium trivially converge almost everywhere to Y.

Converge in terms of the consumption profile follows from the fact that as $m_n^J \to \infty$ for all j as $n \to \infty$, then for any subset of brands Z with positive measure, the probability a random buyer i has a brand in this subset in her consideration set converges to 1. Therefore, the expected valuation of the most preferred brand of each buyer's consideration set converges to 1. As prices converge to the same price almost everywhere, the probability that the utility of a buyer's chosen product is arbitrarily close to 1 (if the buyer chooses to trade) converges to 1. Therefore, the utility of the products consumed by buyers in equilibrium converges in probability to the utility in competitive equilibrium (which is 1). The definition of v_{ij} (from [eq: v_ij]) then implies that the assignment of buyers to sellers in equilibrium is such that the expected distance in the unit interval from the buyer i to its assigned seller j converges to zero. □

Proof of Proposition 3

Proof. Note that sellers can post prices above 0 and sell at least zero, making profits greater or equal to zero. Therefore, negative prices are not consistent with equilibrium. Also, posted prices strictly greater than 1 will always yield zero profits for a seller. However, as $\pi^0(\widehat{m}) > 0$ for any $\widehat{m} \in \mathbb{R}_+$, each seller always has captive buyers, as G is continuous that implies that profits per customer for a price $p \in (0,1)$ are at least as large as

$$\pi^0(\widehat{m}) > 0 \tag{25}$$

Thus, posting a price higher than 1 is inconsistent with equilibrium. Therefore, any candidate equilibrium price must be in [0,1].

Consider a candidate symmetric equilibrium price $\tilde{p} \in [0,1]$. Note that if customer bases satisfy $m^j < q, \forall j$ then profits can be written as

$$\Pi_j(p, \tilde{p}) = m^j \Pi(p, \tilde{p}) \tag{26}$$

where $\Pi(p, \tilde{p}) = pP(p, \tilde{p})$ is the profit margin per customer if the supply constraint is not binding. That is, profits are proportional to the size of the customer base. If the customer bases are identical across sellers, then profits for posting the same price are also identical. By slight abuse of notation, let $\Pi(p, \tilde{p})$ denote the profits of a seller posting p if all sellers have the same customer base $\widehat{m}$.

Note that in both the cases where $m^j < q, \forall j$ and where $m^j = \widehat{m}, \forall j$, the best response of a seller j to a price $\tilde{p}$ is "symmetric," that is, the best response is the same for every seller. Therefore, let $\Phi: [0,1] \to [0,1]$ be the best response correspondence (which is symmetric for all sellers, so we can drop the j superscript), given by

$$\Phi(\tilde{p}) = arg \max_{p \in [0,1]} \Pi(p, \tilde{p}) \tag{27}$$

As a fixed point p^* of $\Phi: [0,1] \to [0,1]$ is the best-response of a best-response to all sellers, it is a symmetric equilibrium. Therefore, to show that a symmetric equilibrium exists, it suffices to show that a fixed point exists. If Φ is single-valued and continuous, then the Brouwer fixed-point theorem implies that such a fixed point exists. Thus, we need to show that Φ is continuous and single-valued.

Continuity: Consider a seller j who is considering posting a price $p \in [0,1]$. Let be the $\epsilon = \tilde{p} - p$ "undercut factor", let $\overline{G}: \mathbb{R} \to [0,1]$ be given by

$$\overline{G}(z) = \begin{cases} 0 & if\, z < 0 \\ G(z) & if\, z \in [0,1] \\ 1 & if\, z > 1, \end{cases} \tag{28}$$

note the probability that a customer i prefers (j,p) over an arbitrary $(h,\tilde{p})$ is given by the probability that $v_{ij} \geq v_{ih} - \epsilon$, which is

$$P\left(v_{ij} \geq v_{ih} - \epsilon\right) = \int_0^1 \int_0^1 I_{\{x+\epsilon \geq y\}}(x,y)dG(y)dG(x) \\ = \int_0^1 \overline{G}(x+\epsilon)dG(x). \tag{29}$$

Since a^j is the same for all sellers, $F(z) = z$, and all other sellers post the same price, the equation of probability of sale to a customer [eq: analytical form] can be re-written as

$$\begin{aligned} P(p,\tilde{p}) &= \int_0^1 I_{\{x-p\geq 0\}}(x)exp\left[-\widehat{m}\left(1-\int_0^1 I_{\{x+\epsilon\geq y\}}(x,y)dG(y)\right)\right]dG(x) \\ &= \int_0^1 I_{\{x-p\geq 0\}}(i)exp\left[-\widehat{m}\left(1-\overline{G}(x+\epsilon)\right)\right]dG(x) \\ &= \int_p^1 exp\left[-\widehat{m}\left(1-\overline{G}(x+\epsilon)\right)\right]dG(x), \end{aligned} \tag{30}$$

as v_{ih} are distributed according to G for all h. Since G is continuous, $P(p,\tilde{p})$ is continuous, which implies that $\Pi(p,\tilde{p})$ is continuous. By the Maximum Theorem, Φ is upper hemicontinuous, nonempty, and compact valued. Therefore, if Φ is a function, then it is continuous.

Single-valued: To show Φ is single-valued (that is, a function), it suffices to show that $\Pi(p,\tilde{p})$ is a strictly concave function of p, note that

$$\Pi(p,\tilde{p}) = pmin\{P(p,\tilde{p}),q\} \tag{31}$$

so it suffices to show that $P(p,\tilde{p})$ is a concave function of p. Equations 28 and 30 imply that, as G is concave, then $\overline{G}$ is concave on $[0,+\infty)$, and therefore p is concave on prices in [0,1]. Therefore, $\Pi(p,\tilde{p})$ is a strictly concave function of p. □

Proof of Proposition 4

Proof. Case 1: The supply constraint q is not binding in equilibrium.

Let $\Phi\colon [0,\overline{p}] \rightrightarrows [0,\overline{p}]$ be the best response correspondence when the supply constraint is not binding, it satisfies

$$\Phi(p) = arg\max_{p\in[0,\overline{p}]} pP(p,p') \tag{32}$$

Clearly, a price $p^* = \Phi(p^*)$ is a symmetric Nash equilibrium.

To characterize Φ, note that equation [eq: analytic probability of sale] implies that the derivative of P^n at the point $p = p'$ does not exist, but there are left and right derivatives, and they satisfy

$$\begin{aligned} \frac{\partial_- P^n(p,p')}{\partial p}\bigg|_{p=p'} &= 0, \\ \frac{\partial_+ P^n(p,p')}{\partial p}\bigg|_{p=p'} &= -\frac{2}{\lambda}. \end{aligned} \tag{33}$$

Note that the left derivative is larger than the right derivative. This implies that the gain in the probability of sale from a marginal decrease in price at p' is lower than the loss from a marginal increase in price. In an interior equilibrium $p = p'$, thus, the left and right derivatives imply that in an interior solution, the marginal change in profits with an increase in price is zero and that the marginal change in profits with a decrease in price is negative.

As P^n is continuously left differentiable, then ξ is continuously left differentiable. Note that an interior solution p^* to 32 satisfies

$$\frac{\partial_+ \Pi(p,p')}{\partial p}\bigg|_{p=p^*} = 0 \tag{34}$$

as the right-hand side derivative always exists for ξ and is smaller than the left-hand side if $p = p'$. This implies that if the return in profits from decreasing the posted price when it is above the competitors' price is zero, then the returns from decreasing the price below the competitors' is strictly negative.

Solving this first order condition for p^* yields the interior solution

$$p^* = \frac{\lambda}{2m} \tag{35}$$

Therefore, Φ is single-valued for interior best responses.

It remains to rule out corner solutions that might occur together with an interior solution, which occurs if a is not too large or small. Posting 0 yields zero profits while posting $p \in (0,\overline{p})$ always yields strictly positive profits, so 0 cannot be a best response. It only remains to find conditions such that the monopoly price $\overline{p}$ is not a best response when Equation 4.8 is satisfied for $p' = p^*$.

In the case where

$$\frac{\partial\_\Pi(p,\overline{p})}{\partial p}\Big|_{p=\overline{p}} \geq 0 \tag{36}$$

the equilibrium price is clearly the monopoly price, which occurs if a is relatively small.

On the other hand, the case where

$$\frac{\partial\_\Pi(p,\overline{p})}{\partial p}\Big|_{p=\overline{p}} < 0 \tag{37}$$

can be problematic since it implies that the monopoly price is not an equilibrium price, but it is not necessarily true that a price p^* satisfying Equation 34 with $p' = p^*$ is also an equilibrium.

To rule out this case consider a candidate interior equilibrium $p^* \in (0,\overline{p})$ that satisfies Equation 34, consider the profits from posting p^* compared to profits from posting the monopoly price

$$\begin{aligned} \Pi(p^*,p^*) &= mp^*P(p^*,p^*) \\ &= \frac{\lambda}{2m}[1-exp(-m)], \\ \Pi(\overline{p},p^*) &= m\overline{p}P(\overline{p},p^* \\ &= \begin{cases} \left(1-\frac{\lambda}{2}\right)exp(-2m) & if\, \overline{p}-p^* > \lambda/2 \\ \left(1-\frac{\lambda}{2}\right)[exp(2\epsilon m)-exp(-m)]. & if\, \overline{p}-p^* \leq \lambda/2 \end{cases} \end{aligned} \tag{38}$$

If $\overline{p} - p^* \leq \lambda/2$ then the marginal gain in lowering prices is higher than posting the monopoly price $\overline{p}$, then trivially profits from posting p^* are higher than $\overline{p}$. Therefore, if p^* is close enough to $\overline{p}$, it is an equilibrium.

If $\overline{p} - p^* > \lambda/2$ then a sufficient condition for p^* to be a Nash equilibrium is that a that satisfies

$$\frac{\lambda}{2m}[1-exp(-m)] > \left(1-\frac{\lambda}{2}\right)exp(-2m) \tag{39}$$

Let m_H that satisfies

$$\frac{\lambda}{2m_H}[1 - exp(-m_H)] = \left(1 - \frac{\lambda}{2}\right) exp(-2m_H) \tag{40}$$

for $m > m_H$ a unique Nash equilibrium exists given by Equation 35.

Let m_L such that $m = m_L$ implies that $\overline{p} - p^* = \lambda/2$. Solving for m_L,

$$m_L = \frac{\lambda}{2(1-\lambda)} \tag{41}$$

Then, for $m < m_L$, the symmetric Nash equilibrium exists and is unique, given by

$$p^* = min\left\{\overline{p}, \frac{\lambda}{2m}\right\} \tag{42}$$

Case 2: *The supply constraint q is binding.*

The quantity sold by sellers must be equal to or lower than q in equilibrium. Therefore, the sellers post prices equal to or above the monopoly price to discourage potential customers from purchasing.

To compute demand for $p^* > \overline{p}$, then the quantity sold by a seller in equilibrium is given by

$$mP(p^*, p^*) = P\left(v_{ij} - p^* \geq 0 | v_{ij} \geq v_{ij'}, \forall j' \in A^i\right)[1 - \pi^0(a)] \tag{43}$$

where $P\left(v_{ij} - p^* \geq 0 | v_{ij} \geq v_{ij'}, \forall j' \in A^i\right)$ is the probability that j's contract is preferred by i over not trading conditional on it being preferred over all other contracts that i is aware of. Let $\overline{\delta}(p)$ be the distance between i and j that makes i indifferent between trading with j and not trading.[10]

Let F^n be a cumulative distribution function that describes the probability that the valuation of the highest valuation buyer is above x conditional on the buyers being aware of n different sellers. As sellers are distributed uniformly F^n is given by

$$F^n(x) = 1 - \left[1 - \left(\frac{1-x}{\lambda}\right)\right]^n \tag{44}$$

[10] Given by $\overline{\delta}(p) = \frac{1-p}{\lambda}$

Then,

$$P\left(v_{ij} - p^* \geq 0 | v_{ij} \geq v_{ij'}, \forall j' \in A^i\right) \quad \begin{aligned} &= \textstyle\sum_{n=0}^{\infty} \pi^n (a) F^{n+1}(p^*) \\ &= 1 - exp\left(-\frac{1-p^*}{\lambda} m\right). \end{aligned} \tag{45}$$

In equilibrium with binding supply constraint, the equilibrium price satisfies p^*

$$mP(p^*, p^*) \quad = q \tag{46}$$

solving for p^* yields

$$p^* = 1 - \frac{\lambda}{m}\Big(log\big(1 - exp(-m)\big) - log(1 - exp(-m) - q)\Big) \tag{47}$$

Proof of Lemma 2

Proof. Consider a symmetric equilibrium price $p^* \in (0,1)$. Given a cost of sampling a seller c, the payoff from sampling n sellers is

$$U(c, n) = S(n) - n \times c \tag{48}$$

where

$$S(n) \quad = E\left[max\{\max_{j=1,\dots,n} v_{ij} - s(j), 0\} | v_{ij} \sim G\right] \tag{49}$$

is the expected surplus obtained from transacting with the highest valuation brand conditional on a search sample size of n. It satisfies

$$S(n) = E[max\{v^n - s(j), 0\} | v^n \sim G^n] \tag{50}$$

where G^n is the distribution of

$$\begin{aligned} G^n(x) \quad &= Prob\left(\max_{j=1,\dots,n} v_{ij} \leq x\right) \\ &= Prob(v_{i1} \leq x, v_{i2} \leq x, \dots, v_{in} \leq x) \\ &= Prob(v_{i1} \leq x) \times Prob(v_{i2} \leq x) \times \dots \times Prob(v_{in} \leq x) \\ &= G(x) \times G(x) \times \underbrace{\quad \dots \quad}_{n-1 \text{ times}} \times G(x) \\ &= [G(x)]^n, \end{aligned} \tag{51}$$

which follows from valuations being independently drawn from different sellers.

Therefore, right hand side of Equation 50 can be written as

$$S(n) = \int \left\{\int_{s(j)}^{1} [x - s(j)]\, d[G(x)]^n\right\} m^j\, dj \tag{52}$$

Since $\int m^j s(j) dj < 1$ and G is continuous, Equation 52 implies that $S(n) > 0$ for every $n \in \mathcal{N}$, that $S(n)$ is strictly increasing on n and $S(n+1) - S(n) < S(n) - S(n-1)$.

Given a search cost c, the optimal sample size k for a buyer satisfies

$$S(k+1) - S(k) \geq c \geq S(k) - S(k-1) \tag{53}$$

therefore let $c^k \in [S(k) - S(k-1), S(k+1) - S(k)]$ for each $k \geq 1$, and c^0 such that $c^0 > S(1)$. □

Proof of Proposition 5

Proof. Lemma 2 shows that given a pricing strategy $s: [0,1] \to \mathcal{R}_+$ such that $\int m^j s(j) dj < 1$ there is a cost of a randomly sampling a seller $c^k > 0$ such that sampling k sellers is optimal. Then, if customer types by search costs are distributed according to a Poisson probability mass function π with parameter $\widehat{m} = \int m^j dj$, where $\pi(k)$ is the fraction of customers with search costs c^k, $(\mathbf{m}, p^*)$ are consistent with optimal customer search. □